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Exam I , MTH 221 , Fall 2010, 2pm section

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QUESTION 1. **(30 points)** Given $A^{-1} = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 3 & 0 \\ -2 & 2 & 3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & -2 & -2 \\ -4 & 2 & 2 \\ 0 & 1 & -2 \end{bmatrix}$

(i) Find the 2nd row of $(AB)^{-1}$ = $B^{-1}A^{-1}$

$-4[2\ \ -2\ \ -2] + 2[-2\ \ 3\ \ 0] + -2[-2\ \ 2\ \ 3] = [-8\ \ 10\ \ 2]$ ✓

(ii) Find the third column of $(BA)^{-1}$ = $A^{-1}B^{-1}$

$-2\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix} + -2\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ 2 \end{bmatrix}$ ✓

(iii) Find the (2, 3)-entry of $B^{-1}A^{-1}$ ⇒ 2nd row of B^{-1}, 3rd col. of A^{-1}

$[-4\ \ 2\ \ 2] \times \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = (-4)(-2) + 2(0) + 2(3) = 8 + 6 = 14$

(iv) Solve the system $AX = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ given A^{-1} ⇒ $A^{-1}AX = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 3 & 0 \\ -2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ ($\to A^{-1}$)

I_3

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 3 & 0 \\ -2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ → we'll get 3x1

3x3 3x1

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix} + -1\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \\ -1 \end{bmatrix}$

$x_1 = 4$

$x_2 = -11$

$x_3 = -1$ ✓

(v) Find the matrix A

QUESTION 2. (12 points) Let $D = \begin{bmatrix} 4 & 2 & b & a \\ -4 & -2 & -2 & 2 \\ 2 & 2b & 0 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

(i) For what values of a, b does the system $DX = \begin{bmatrix} 5 \\ -5 \\ 7 \\ 0 \end{bmatrix}$ have a unique solution?

(ii) For what values of a, b will D be singular(non-invertible)?

QUESTION 3. (30 points) Let A be a 4×4 matrix. Given

has 3det(A) has 15det(A)

$$A \underbrace{\ 3R_1\ } A_1 \underbrace{\ R_3 \leftrightarrow R_2\ } A_2 \underbrace{\ -5R_2\ } A_3 \underbrace{\ -4R_1 + R_2 \to R_2\ } A_4 = \begin{bmatrix} -4 & 2 & 2 & -4 \\ 0 & 5 & 3 & -2 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

has −3det(A) has 15det(A)

det(A4) = (−4)(5)(3)(2) = −120

(i) Find $det(A)$.

det(A4) = 15det(A)
−120 = 15 det(A) ⇒ det(A) = −8

(ii) Find $det(-2A_2)$ = $(-2)^4 det(A_2) = 16(-3det(A)) = 16(-3)(-8) = 384$

(iii) Find a matrix B such that $BA = A_2$ 4x4 use I_4

$$\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix} \xrightarrow{3R_1} \begin{bmatrix} 3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \overbrace{\begin{bmatrix} 3&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1 \end{bmatrix}}^{B}$$

$$\therefore \begin{bmatrix} 3&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1 \end{bmatrix} A = A_2$$

(iv) Find $det(A^2A_2)$ = $det(AAA_2) = det(A)det(A)det(A_2) = (-8)(-8)(-3)(-8)$ $= 1536$

(v) Find $det((A^{-1}A_4)^T)$ = $det(A_4^T A^{-1\,T}) = det(A_4^T)det(A^{-1\,T}) = det(A_4)det(A^{-1})$
$= (-120)\left(\frac{1}{det(A)}\right) = (-120)\left(\frac{1}{-8}\right) = 15$

let $C = A_4$

(vi) Find the (3,4)-entry of A_4^{-1} = $\frac{C_{43}}{det(A_4)} = \frac{(-1)^7 |C_{43}|}{det(A_4)} = \frac{-\begin{vmatrix} -4 & 2 & -4 \\ 0 & 5 & -2 \\ 0 & 0 & 2 \end{vmatrix}}{-120}$

sp. case

$$\frac{-\begin{vmatrix} -4 & 2 & -4 \\ 0 & 5 & -2 \\ 0 & 0 & 2 \end{vmatrix}\begin{matrix} -4 & 2 \\ 0 & 5 \\ 0 & 0 \end{matrix}}{-120} = \frac{-[(-40)+0+0]-[0+0+0]}{-120}$$

$$= \frac{40}{-120} = \frac{-1}{3}$$

QUESTION 4. (12 points) (b) Find a 2×2 matrix A such that $\begin{bmatrix} 7 & 7 \\ 1 & 5 \end{bmatrix} A + 3I_2 = 3A + \begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}$

QUESTION 5. (16 points) Solve the following system:

$x_3 + x_4 - x_5 = 1$
$x_1 + x_2 - x_3 + x_5 = 2$
$2x_1 + 3x_2 + x_3 - 2x_5 = 10$

Then give me one numerical solution to the system.

Faculty information

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